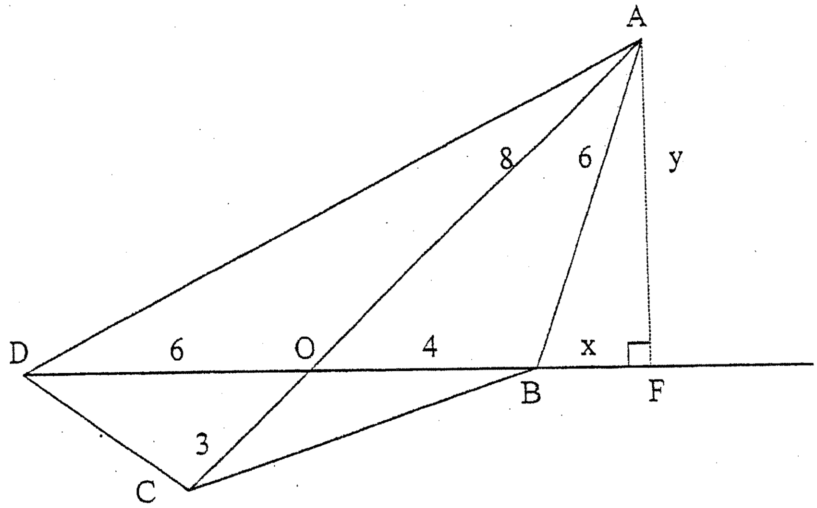
教育部九十學年度高級中學數學科能力競賽複賽

90 學年度台南區數學競試（II）【參考解答】

1.



$$\begin{cases} x^2+y^2=36 \\ (x+4)^2+y^2=64 \end{cases} \Rightarrow\ x=\frac{3}{2}\ ,\ y^2=\frac{135}{4}$$

$$\therefore\ AD=\sqrt{(10+x)^2+y^2}=\sqrt{166}$$

2.

$$\frac{3\sin x+\sin 3x}{3\cos x+\cos 3x}=\frac{\tan x(1+2\cos^2 x)}{2\cos^2 x}=\frac{\tan x}{2}(\sec^2 x+2)=\frac{\tan x}{2}(\tan^2 x+3)=\frac{\sqrt{5}}{2}(5+3)=4\sqrt{5}$$

3.

證明：

$x_n > 0$ and $\sum_{n=1}^{2000} x_n = 1 \quad \Rightarrow \quad 0 < x_n < 1, \ \forall n. \quad \Rightarrow \quad 0 < 1 - x_n < 1 \ \ \forall n.$

$$\therefore \ (1) \quad \sum_{n=1}^{2000} \frac{1}{1-x_n} > \sum_{n=1}^{2000} \frac{1-{x_n}^2}{1-x_n} = \sum_{n=1}^{2000} (1+x_n) = 2000 + \sum_{n=1}^{2000} x_n = 2001$$

$$\text{or} \ (2) \quad \sum_{n=1}^{2000} \frac{1}{1-x_n} = \sum_{n=1}^{2000} \left(1 + \frac{x_n}{1-x_n}\right) = 2000 + \sum_{n=1}^{2000} \frac{x_n}{1-x_n} > 2000 + \sum_{n=1}^{2000} x_n = 2000 + 1 = 200$$

4.

$p(5) = p\{$前 4 次錯誤，第 5 次正確$\} = \frac{99}{100} \cdot \frac{98}{99} \cdot \frac{97}{98} \cdot \frac{96}{97} \cdot \frac{1}{96} = \frac{1}{100}$

for n=1,2,...,100

$p(n) = p\{$前 n-1 次錯誤，第 n 次正確$\} = \frac{99}{100} \cdot \frac{98}{99} \cdots \frac{100-n+1}{100-n+2} \cdot \frac{1}{100-n+1} = \frac{1}{100}$

5.

$ax^2 + bx + c = a(x-a)(x-b) = a[x^2 - (a+b)x + ab]$

比較係數：$\begin{cases} b = -a(a+b) \\ c = a^2 b \end{cases} \quad \therefore b + ab = -a^2 \quad$ i.e $\quad b(1+a) = -a^2 \quad \therefore 1+a \mid a^2$

但 $a^2 = (a+1-1)^2 = (1+a)^2 - 2(1+a) + 1$ 且 $1+a \mid a^2$ 故 $1+a \mid 1$

$\therefore a = 0$ or $a = -2$ 但 $a \neq 0$. 所以 $a = -2$

代入 b=2(-2+b) => b=-4+2b => b=4

代入 $c = a^2 b = (-2)^2 \times 4 = 16$

$\therefore \quad a = -2, b = 4, c = 16.$